HISTORICAL EXERCISE SET.

1. As indicated earlier, Egyptian geometers used the formula $A = (1/4)(a+c)(b+d)$ to calculate the area of any quadrilateral whose successive sides have lengths a, b, c, and d.

(a) Does this formula work for squares? For rectangles that are not squares?

(b) If you choose specific lengths for the sides of an isosceles trapezoid, how does the result compare to the actual area? Repeat for two other isosceles trapezoids. Do the same for three specific parallelograms.

(c) Generalize your results for part (b).

2. If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, Babylonian geometers approximated the length of the hypotenuse by the formula $c = b + (a^2/2b)$.

(a) How does this approximation compare to the actual result when $a = 3$ and $b = 4$? When $a = 5$ and $b = 12$? When $a = 12$ and $b = 5$?

(b) Give an algebraic argument demonstrating that this formula results in an approximation that is too large.

3. The following was translated from a Babylonian tablet created about 2600 B.C. Explain what it means.

> 60 is the circumference, 2 is the perpendicular, find the chord. Double 2 and get 4, do you see? Take 4 from 20 and get 16. Square 20, and you get 400. Square 16, and you get 256. Take 256 from 400 and you get 144. Find the square root of 144. 12, the square root, is the chord. This is the procedure.

4. The Moscow Papyrus (c. 1850 B.C.) contains the following problem:

> If you are told: A truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top. You are to square this 4, result 16. You are to double 4, result 8. You are to square 2, result 4. You are to add the 16, the 8, and the 4, result 28. You are to take one third of 6, result 2. You are to take 28 twice, result 56. See, it is 56. You will find it right.

Show that this is a special case of the general formula, $V = (1/3)h(a^2 + ab + b^2)$, for the volume of the frustum of a pyramid whose bases are squares, whose sides are a and b, respectively, and whose height is h. Also, use calculus to derive the formula mentioned above.

5. An Egyptian document, the Rhind Papyrus (c. 1650 B.C.), states that the area of a circle can be determined by finding the area of a square whose side is 8/9 of the diameter of the circle. Is this correct? What value of π is implied by this technique?

6. It is said that Thales indirectly measured the distance from a point on shore to a ship at sea using the equivalent of angle-side-angle (ASA) triangle congruence theorem. Make a diagram that could be used to accomplish this feat.

7. Eratosthenes (c. 275 B.C.), a scholar and librarian at the University at Alexandria, is credited with calculating the circumference of the earth using the following method: Eratosthenes observed that on the summer solstice the sun was directly overhead at noon in Syene (the present site of Aswan), while at the same time in Alexandria, which was due north, the rays of the sun were inclined 7°12', thus indicating that Alexandria was 7°12' north of Syene along the earth's surface. Using the known distance between the two cities of 5000 stades (approximately 530 miles), he was able to approximate the circumference of the earth. Make a diagram that depicts this method and calculate the circumference in stades and in miles. How does this result compare to present-day estimates?