Math 1710 Quiz II  
Fall 2006  

Follow the directions for each problem. Show all work and justify your answers CREDIT CANNOT BE GIVEN FOR UNJUSTIFIED ANSWERS.

1. (5 points) Find the value of $a$ which makes the function

$$g(x) = \begin{cases} 
  x^2 - a^2 & x \neq a \\
  8 & x = a 
\end{cases}$$

continuous for all real numbers.

Since $g(x)$ matches the function $f(x) = \frac{x^2 - a^2}{x - a}$ everywhere except at $x = a$, and since $f(x)$ is a rational function, we know that $f(x)$ (and therefore $g(x)$) is continuous everywhere except where the denominator is 0. As the denominator is 0 only at $x = a$, we see that $f(x)$ (and therefore $g(x)$) is continuous everywhere except at $x = a$. We now wish to choose a value of $a$ so that $g(x)$ is continuous at $x = a$. In order for that to happen, it must be the case that $\lim_{x \to a} g(x) = g(a) = 8$, so we need to calculate $\lim_{x \to a} g(x)$ and choose $a$ so that the limit is 8.

Now,

$$\lim_{x \to a} g(x) = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \lim_{x \to a} \frac{(x - a)(x + a)}{x - a} = \lim_{x \to a} x + a = 2a,$$

so we need $2a = 8$. Thus, if we take $a = 4$, $g(x)$ is a continuous function for all real numbers.

2. (5 points) A patrol car is parked 50 feet from a long warehouse. The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolutions per second. The rate $r$ at which the light beam moves along the wall is

$$r = 50\pi \sec^2 \theta \text{ ft/sec}.$$

Find $\lim_{\theta \to \left(\frac{\pi}{2}\right)^-} r$.

Here we need to know

$$\lim_{\theta \to \frac{\pi}{2}^-} r = \lim_{\theta \to \frac{\pi}{2}^-} 50\pi \sec^2 \theta = 50\pi \lim_{\theta \to \frac{\pi}{2}^-} \frac{1}{\cos^2 \theta}.$$

If we plug $\frac{\pi}{2}$ in for $\theta$ (which normally would be legal since $\cos \theta$ is a continuous function as is $\frac{1}{\theta}$), we get $\frac{1}{0}$, where, by “+0,” I mean that as we approach $\frac{\pi}{2}$ from the left, we get small positive numbers in the denominator. Since the numerator is nonzero, and the denominator is always positive as we approach $\frac{\pi}{2}$ from the left, this limit is $+\infty$.

See next page
3. (5 points) Use the definition of the derivative to calculate the derivative of $y = \sqrt{x + 2}$ and then find the equation of the tangent line to $y = \sqrt{x + 2}$ at $x = 7$.

\[
f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}
\]

\[
= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 2) - (x + 2)}{\Delta x(\sqrt{x + \Delta x + 2} + \sqrt{x + 2})}
\]

\[
= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x + 2} + \sqrt{x + 2})}
\]

\[
= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}
\]

\[
= \frac{1}{2\sqrt{x + 2}}
\]

When $x = 7$ therefore, the slope of the tangent line is $f'(7) = \frac{1}{2\sqrt{7 + 2}} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$.

Also, the point of tangency is $(7, f(7)) = (7, \sqrt{9}) = (7, 3)$. Thus, the equation of the tangent line is

\[
y - 3 = \frac{1}{6}(x - 7).
\]